Discrete Math Rules of Logic Test                    Name ___*José Ejemplo*___

1.  Determine if the statements $P \to (Q \vee R)$ and $(P \to Q) \vee (P \to R)$ are logically equivalent.

    From the following truth table (I cheated and let Python make it for me ;-):

```
$ ttg_cli.py "['P', 'Q', 'R']" -p "['P => (Q or R)', '(P => Q) or (P => R)']" -i False

+-------+-------+-------+----------------+------------------------+
|   P   |   Q   |   R   |  P => (Q or R) |  (P => Q) or (P => R)  |
|-------+-------+-------+----------------+------------------------|
| True  | True  | True  |      True      |          True          |
| True  | True  | False |      True      |          True          |
| True  | False | True  |      True      |          True          |
| True  | False | False |      False     |          False         |
| False | True  | True  |      True      |          True          |
| False | True  | False |      True      |          True          |
| False | False | True  |      True      |          True          |
| False | False | False |      True      |          True          |
+-------+-------+-------+----------------+------------------------+
```

    we see that the two statements have the same truth values, and they are therefore logically equivalent by definition.


2.  Decide whether the following is a valid deduction rule:

$$P \to Q$$
$$P \to R$$
$$\overline{\therefore \ P \to (Q \wedge R).}$$

```
ttg_cli.py "['P', 'Q', 'R']" -p "['P => Q', 'P => R', 'P => (Q and R)']" -i False

+-------+-------+-------+----------+----------+-----------------+
|   P   |   Q   |   R   |  P => Q  |  P => R  |  P => (Q and R) |
|-------+-------+-------+----------+----------+-----------------|
| True  | True  | True  |   True   |   True   |      True       |
| True  | True  | False |   True   |   False  |      False      |
| True  | False | True  |   False  |   True   |      False      |
| True  | False | False |   False  |   False  |      False      |
| False | True  | True  |   True   |   True   |      True       |
| False | True  | False |   True   |   True   |      True       |
| False | False | True  |   True   |   True   |      True       |
| False | False | False |   True   |   True   |      True       |
+-------+-------+-------+----------+----------+-----------------+
```

    The last column, the conclusion, is true whenever both hypotheses are true, and false otherwise, so this is a valid deduction rule.

3. Tommy Flanagan was telling you what he ate yesterday afternoon. He tells you, "I had either popcorn or raisins. Also, if I had cucumber sandwiches, then I had soda. But I didn't drink soda or tea." Of course, you know that Tommy is the world's worst liar, and everything he says is false. What did Tommy eat? Justify your answer by writing all of Tommy's statements using sentence variables (P, R, C, S and T), taking their negations, and using these to deduce what Tommy actually ate.

Using the following sentence variables:

P: Tommy ate popcorn.
R: Tommy ate raisins.
C: Tommy ate cucumber sandwiches.
S: Tommy drank soda.
T: Tommy drank tea.

Tommy's statements can be written:

$P \lor R$
$C \to S \equiv \neg C \lor S$
$\neg(S \lor T) \equiv \neg S \land \neg T$

Since we know Tommy is an inveterate liar, we need to negate all his assertions to get at the truth.

$\neg P \land \neg R$
$C \land \neg S$
$S \lor T$

All of these statements must be true, so Tommy had cucumber sandwiches and tea.